Introducing Multi-Criteria Path Planning with Terrain Visibility Constraints: The Optimal Searcher Path Problem with Visibility

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1 Introduction

We are interested in the *optimal searcher path* problem *with visibility* (OSPV problem) [1], a novel path planning approach to detection search introducing the notion of inter-region visibility in the classical *optimal searcher path* (OSP) [2] problem from search theory [3]. Usual solving techniques of OSP-related problems involve Branch and Bound algorithms (*e.g.*, [4]) and dynamic programming (*e.g.*, [5]). We developed a *mixed-integer linear program* (MILP) along with an *ant colony optimization* (ACO) algorithm for the single criterion OSPV case. Then, we extended the OSPV problem to use multiple criteria and introduced two *ant colony optimization* (ACO) [6] algorithms. Section 2 briefly introduces the OSPV problem, section 3 outlines our solving techniques and section 4 concludes.

2 The Optimal Searcher Path Problem with Visibility

Given an environment discretized by a set R of N regions and a total number of time steps T, the objective is to find a search plan for a constrained search unit (searcher) maximizing the probability of finding a search object (moving or not) by the allocation of the available search effort; a discrete quantification in Q units of the resources available for searching at each time step. A search plan P consists in the unit's path (defined by $y_t \in R$ for each time $t \in \{1,...,T\}$ with y_0 being the unit's initial position) and in a sequence of effort allocations to the regions of R (defined by $e_t : R \rightarrow \{0,...,Q\}$ for each time $t \in \{1,...,T\}$). At each time step, the unit chooses its next destination respecting an accessibility map $A : R \rightarrow 2^R$ and allocates the Q effort's units to a subset of visible regions respecting a visibility map $V : R \rightarrow 2^R$. The object's location is *a priori* unknown and is characterized by an initial probability of containment distribution on R (the poc_0 distribution). The object's Markovian motion model is characterized by a matrix d where $d(s,r)$ is the probability of an object's move from region s to region r within one time step. Usually, each row of d forms a distribution. The unit's as limited detection capabilities characterized as a function of its current position s, of the amount of discrete effort q allocated to a destination region r and of time t. This conditional probability of detection (*pod*) is defined as

$$\forall t \in \{1,...,T\}, \forall s, r \in R, \forall q \in \{0,...,Q\} : pod_t(s,r,q) = 1 - \exp(-W_t(s,r) \times q), \quad (1)$$

where W is the detectability index of the object defined in function of the distance between s and r, of the area of r and of time t. The *pod* is conditional to the object's presence in region r at time t and is used to compute the probability of success (*pos*) in a region r at a time t that is conditional to the non-detection of the object up to time t (2).

$$\forall t \in \{1,...,T\} : \forall r \in R : pos_t(r) = poc_t(r) \times pod_t(y_t, r, e_t(r)), \quad (2)$$

with $pos_0(r) = 0$ for all regions r. The *poc* at time t, conditional to the non-detection of the object up to time t, varies according to the object's motion model and to the unit's effort allocations (3).

$$\forall t \in \{1,...,T\} : \forall r \in R : poc_t(r) = \sum_{s \in R} d(s,r) \left[poc_{t-1}(s) - pos_{t-1}(s) \right]. \quad (3)$$

As mentioned, the goal is to maximize the probability of finding the object or the cumulative overall probability of success of a plan P defined as $COS(P) = \sum_{t \in \{1,...,T\}} \sum_{r \in R} pos_t(r)$. As a matter of fact, search operations (*e.g.*, in search and rescue) do not consist in finding the object's at all costs and we defined two supplementary criteria: the plan's complexity and the unit's safety. We quantified the complexity in function of total path length (*TPL*) and the unit's safety using the cumulative probability of hazard (*CH*) defined as $CH(P) = 1 - \prod_{t \in \{1,...,T\}} (1 - poh(y_t))$ where $poh(r)$ is the independent probability that something undesirable happens to the unit in region r at any search plan's step.

3 Solving Single Criterion and Multi-Criteria OSPV Problems

Since the *pod* function, the poc_0 distribution and the object's motion model are known data and since the *pod* function varies as a function of a discrete search effort, the single criterion OSPV problem can be reformulated as a MILP. The model, solved by the generic solver ILOG CPLEX 11.2, is used in [1] as a point of comparison for the *ant search* (AS) algorithm; our adaptation of ACO to the problem. In ACO, a colony of candidate solutions is built at each iteration (one solution per "ant"). A subset of plans from this colony is used to update the *pheromone trails* acting as a common memory to stochastically guide the ants' construction choices, *i.e.*, where to move and to search. These stochastic choices' probabilities are guided by the *transition probabilities* and the trails are updated proportionally to the solution or component's quality. Usually, an evaporation factor decreases the trails at each iteration to avoid stagnation. AS uses 2 $T \times N$ pheromone tables, one for the unit's moves and one for its allocations, and defines its transition probabilities as $v_{act} / \sum v_{act}$ where v_{act} is the pheromone value of the feasible action *act*. The algorithms were tested on 8 randomly generated OSPV square grid environments. The largest, $N = T = 225$, $Q = 5$, involves a solution's space with more than 10^{500} possible search plans. The smallest one, $N = T = 4$, $Q = 5$, involves a total of more than 10^8 possible search plans.

In the multi-criteria case, our algorithms' goal is to approximate the Pareto optimal set (*PSET**) defined as the set of all non-dominated solutions. Our first algorithm, PAS [1], is a Pareto ACO (PACO) adaptation to the OSPV problem. PACO, developed by Doerner *et al.*, has been successfully applied to a variety of multi-criteria problems (*e.g.*, [7]). LAS [1], our second algorithm, is similar to PAS in that its goal is to approximate *PSET**. The main difference is in the update process. While PAS simultaneously considers all the criteria when updating the trails belonging to one action type, LAS stores a permutation of criteria corresponding to the current priority order of criteria and varies this priority to explore different solutions space's sub-spaces. Convergence-based and diversity-based indicators from [8] were used to evaluate the algorithms.

4 Discussion and Conclusion

While the MILP model guarantees optimality if it has enough time and enough resources in the single criterion case, our current results show that refinements are needed to handle large OSPV environments with ILOG CPLEX since the larger models do not fit into memory for our current configuration (Intel Core2 Quad Q6600 CPU with 3 GB of RAM); environments up to $N = T = 49$, $Q = 5$ has been loaded successfully by ILOG CPLEX. On the other side, AS performed well in finding solutions in relatively short time (see [1]). For the problems fitting into memory using ILOG CPLEX, the average ratio of the *COS* value by the time (in seconds) to obtain the last incumbent is of 0.0 for our MILP solving scheme and of 1.4 for AS. In the multi-criteria case, the two algorithms (PAS and LAS) performance was similar. LAS has a tendency at better diversification while PAS has a tendency to converge faster (see [1]). At our knowledge, ACO techniques have never been applied to search theory problems and this is the first attempt at solving a multi-criteria OSP problem extension. We strongly believe that the introduced concept of inter-region visibility will provide benefits in allowing lower level search plans that could benefit both manned and unmanned patrols in considering terrain's features (*e.g.*, obstacles, topology) and operational features (*e.g.*, direct threats, units constraints).

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